Abstract—In an urban environment the differential phase due to thermal expansion of structures is relevant. Uncorrected, the thermal expansion phase leads to a loss of persistent scatterers and increased errors in the deformation time series. In the upper part of tall buildings the thermal expansion phase can vary strongly over time which may result in a complete lack of displacement information if uncompensated. The objective of our work was to estimate and mitigate the thermal expansion phase in our PSI processing. As a result it became possible to include tall buildings into the PSI solution and the accuracy of the solution was improved for all scatterers affected by thermal expansion.

I. INTRODUCTION

Considering the thermal expansion coefficient of concrete (roughly 0.00001 m/mK) we can calculate that a 100m high building will change its height by about 1mm per deg. temperature change. This thermal expansion is expected to affect the SAR interferometric phase. This is particularly relevant when working with shorter wavelengths. Thermal effects affecting Persistent Scatterer Interferometry (PSI) have been observed by several authors [1-4]. A good introduction on this topic is found in [1]. Monserrat et al. suggest extending the interferometric phase model by adding a thermal expansion phase term and to replace the two-dimensional regression used in PSI processing with a three-dimensional regression that also estimates the thermal dilation parameter. Other authors [5,6] modeled the thermal expansion effect as a periodic seasonal phase term.

In our work presented here the objective was to use a physically based approach, similar to [1], but to avoid solving a 3 dimensional regression that also has to solve the phase unwrapping as we have found this may not be computationally efficient and robust. One focus of our work was to achieve PSI solutions also on tall buildings.

II. APPROACH

A. Thermal expansion

To a first approximation, the change in length measurements of an object due to thermal expansion is related to the temperature change by a linear expansion coefficient, $\alpha$:

$$\Delta length = \alpha \cdot L \cdot \Delta temp$$  \hspace{1cm} (1)

With $L$, the length of the object and $\Delta temp$ the temperature change.

B. Thermal expansion InSAR phase model

The InSAR phase difference $\Delta \theta$ caused by the temperature change is proportional to the thermal displacement in the line-of-sight direction:

$$\Delta \theta \propto \alpha \cdot L \cdot \Delta temp$$  \hspace{1cm} (2)

Considering the linear dependence of $\Delta \theta$ on $\Delta temp$ we write

$$\Delta \theta = a + b \cdot \Delta temp$$  \hspace{1cm} (3)

With $b$ including both the length of the object and the linear expansion coefficient, furthermore it includes a direction scaling factor for the projection to the line-of-sight (LOS). The parameter $b$ is the derivative of the interferometric phase w.r.t. the temperature for each point ($\partial \theta / \partial temp$). The unit of $\partial \theta / \partial temp$ is [radians/degree C]. The parameter $a$ is added...
to account for the fact that the regressions typically don’t run through zero phase at on $\Delta temp$ equal to zero.

C. Methodology implemented and assumptions used

One possible implementation of a mitigation strategy is be to update the two dimensional regression with respect to the perpendicular baseline and the time interval to a three dimensional regression with respect also to the temperature difference (Monserrat et al., 2011). We decided against this approach as it results in a significant additional computation effort and a higher uncertainty in the phase unwrapping that is also done in the same processing step. Instead, we preferred to use a methodology where we initially ignore the thermal expansion phase and only address it after having estimated the main other phase components as the topographic phase, the deformation phase and the atmospheric phase. In the present example we used a linear deformation model, but using an approach considering non-uniform motion may also be used. After achieving this we proceed with the estimation of the thermal expansion using the following assumptions:

- $b$ is different for each point
- the temperature is the same for all points in the scene
- other phase components have been estimated and subtracted (except phase noise)

Based on the stack of unwrapped residual phases that only include the thermal expansion phase and phase noise we estimate for each point the slope ($b$) and offset ($a$) in a linear regression. The temperature differences used are calculated based on one temperature per acquisition. Considering the effect of sun-shine, but also the thermal capacity of the construction materials and knowing that the supporting structure is usually not the facade, but a steel or concrete structure within the building it is not fully clear what temperatures to use. For our data acquired in the morning hours we used daily averages rather than air temperatures measured close to the acquisition time.

In addition to the regression parameters we also determined for each interferometric pair the thermal expansion phase model. Subtracting the thermal expansion phase model from the residual phases resulted in a significant reduction of the residual phase over high buildings for pairs with high temperature differences.

Using just a single temperature per acquisition is of course a strong simplification of the real thermal properties. Nevertheless, the results achieved showed that a large fraction of the thermal expansion phase could be mitigated in spite of this simplification, especially if we applied a refinement of the temperature differences based on the data itself. For some interferometric pairs the thermal expansion phases calculated based on the available temperatures (in our case the daily averages that we found on the internet) was not perfect. Modifying the temperature differences permitted improving the modeling of the residual phase. This optimization of the temperature was determined such that it minimizes the residual phase for each interferometric pair.

D. Spatial expansion of solution to tall buildings

One limitation of the described methodology is that it starts from a pre-existing solution that was derived without consideration of the thermal effects and therefore it typically does not include the solution for tall buildings. This problem can be overcome by an iterative spatial expansion. The point heights, deformation rates, atmospheric phases, and thermal expansion phases of the existing solution are spatially extrapolated to its neighboring pixels. Starting the next iteration from this spatially expanded solution will result in an expansion of the area for which a solution is found in the regression – this is particularly the case as the point heights as well as the thermal expansion phases are reasonable approximations for other nearby points on the same building.

III. DATA AND RESULTS

The data set used for the demonstration and verification consists of an interferometric stack of 50 TerraSAR-X stripmap-mode SLCs acquired over the city of Barcelona between 2007 and 2012. An overview of the section considered is shown in Fig. 1.

As temperatures we used daily averages that we found on the internet. The temperatures were then optimized in a refinement step as discussed in Section II.C. In this step the temperatures were changed on average by about 2 degrees with the highest changes being about 8 degrees.

After completing a PSI processing without consideration of thermal effects we started with the approach described in Section II. We included several iterations to achieve a good coverage also on tall buildings. Fig. 2 shows for a selected interferometric pair with significant thermal effects the original residual phase, the final estimated thermal phase and the final residual phase after subtraction of the estimated thermal phase. We notice in the original residual phase that there was no coverage on the higher elevations of the tallest buildings. For the top part of the highest building (height of about 144m) we count more than one full phase cycle of thermal expansion phase. After subtraction of the thermal expansion the residual phase looks overall significantly smoother with smaller values, which clearly indicates an improvement of the quality. Furthermore, coverage was achieved for the tall buildings. Fig. 3 shows $\Delta \theta / \Delta temp$ using two different color scales to better visualize both the extreme values on the tallest building (up to about 0.6 radian/deg. C) as well as the smaller but still relevant effects on the lower buildings.

To quantify the quality we considered the phase deviation from the bi-linear regression with and without subtraction of the thermal expansion phase (Fig. 4). Without subtraction of the thermal expansion phase the phase standard deviation increases with the building height resulting in a rejection of the solutions higher up on the tall buildings. After mitigating
the thermal expansion effects the phase standard deviation does not depend as much on the building height so that solutions are found to the top of the buildings.

The average deformation rate estimated after the mitigation of the thermal expansion effects is shown in Fig. 5. It is shown in slant range geometry, so that the high buildings are more clearly visible. A deformation time series including also the thermal expansion effects is shown in Fig. 6 of a point near the top of the tallest building in Fig. 3. Peak to peak dilatation of more than 2.5cm is observed.

Fig. 1. Averaged backscatter intensity obtained from an interferometric stack of 50 TerraSAR-X data sets showing parts of the city of Barcelona.

(a) residual phase (at start)

(b) estimated thermal expansion phase

(c) residual phase after subtraction of estimated thermal expansion phase

Fig. 2. Example of improvement for one interferometric pair of a TX data stack over a section of Barcelona. One color cycle corresponds to two phase cycles.

Fig. 3. \( \frac{\partial \Phi}{\partial \text{temp}} \) determined for the TX stack over Barcelona, shown using two different color scales: 1rad/1deg C per color cycle (top) and 0.2rad/1deg C per color cycle (bottom).

Fig. 4. Phase standard deviation from bi-linear linear regression without (top) and with (bottom) subtraction of the thermal expansion phase.

Fig. 5. Average deformation rate estimated from TX stack over Barcelona after mitigation of thermal expansion effects.
IV. DISCUSSION AND CONCLUSIONS

We proposed a methodology to estimate in PSI processing a thermal expansion phase term without expanding the regression analysis to a three dimensional regression. This applicability of this methodology could be demonstrated using a TerraSAR-X data stack over an urban area with tall buildings. A quite large stack with 50 scenes, acquired over several years was used. The methodology should also be applicable for small stacks. Nevertheless, a reliable separation of thermal effects and linear motion may be difficult if the two effects are strongly correlated, as is the case for a data set acquired between January and June of the same year.

The estimation and mitigation of a thermal expansion phase term improved the PSI result in three aspects. The first improvement is that the residual phase (phase noise) could be significantly reduced which corresponds to an increase of the quality of the result. The second improvement is that the spatial coverage of the result could be extended to high buildings and that more points were in the solution also for smaller buildings. The third improvement is that a physical interpretation could be given to relevant phase term present. The thermal expansion phase term may actually be more than just a way to reduce the residual phase but it may be of interest as additional information.

The estimated maximum $\partial \theta / \partial \text{temp}$ of about 0.6 radian per Kelvin temperature difference corresponds to a LOS $\partial \text{displacement} / \partial \text{temp}$ of about 0.8mm/K and assuming a vertical displacement to a value close to 1.0mm/K. Considering that the building height is around 144m this value is slightly below the value expected for concrete that would be around 1.44mm/K for this height. Considering that we don’t know exactly the thermal expansion factors for the building and also not exactly the applicable length this is a reasonable correspondence.

Using $\partial \text{displacement} / \partial \text{temp}$ and the difference between the point height and the SRTM DEM height we can estimate the thermal expansion coefficients. In this estimation a displacement in the vertical direction was assumed. This estimation is not stable for small height differences, especially considering the quality of the DEM used, and therefore only shown for height differences larger than 20m (Fig. 7).

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VI. REFERENCES