Phase Unwrapping with GAMMA ISP
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Overview

The main scope of this technical report is to provide documentation on phase unwrapping with the GAMMA ISP Software.

This technical report consists of this introductory page with the overview and the table of contents, the publication "Werner C., U. Wegmüller, and T. Strozzi, Processing strategies for phase unwrapping for INSAR applications, Procs. EUSAR Conf., Cologne, Germany, 4.-6. June, 2002", as a more general discussion on phase unwrapping without specific references to GAMMA software programs, and four appendices with specific reference to GAMMA software on pre-processing, unwrapping using branch-cuts region growing algorithms, unwrapping using an algorithm based on minimum-cost-flow (MCF) optimization in a triangular irregular network (TIN), and post-processing.

Contents

PROCESSING STRATEGIES FOR PHASE UNWRAPPING FOR INSAR APPLICATIONS

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ABSTRACT

One of the most challenging aspects in the successful application of SAR interferometry (INSAR) is unwrapping the interferometric phase. The difficulties arise in attempting to find global optimization procedures with the best possible cost criteria for data that are both noisy and incomplete. Recent progress in this problem includes introduction of network flow optimization, and the use of triangular irregular networks for sparse data. Interferograms differ greatly in the difficulty to unwrap depending on the interferogram fringe complexity and correlation. We examine the characteristics of these types and present phase unwrapping strategies for each of these.

1 INTRODUCTION

The absolute unwrapped interferometric phase derived is directly proportional to the difference in path lengths for the SAR image pair. Applications of interferometry relate the unwrapped phase to geophysical parameters such as elevation, and deformation due to ice motion, surface subsidence, earthquakes, volcanic inflation/deflation, and tectonic motion [1,2]. The complex-valued interferogram values however, are known only modulo 2π and must be unwrapped to obtain a quantitative interpretation.

Phase unwrapping is the process of restoring the correct multiple of 2π to each point of the interferometric phase image.

For a well-behaved smooth phase field all the unwrapped phase differences between adjacent interferogram samples lie between -π and +π. When this is true, phase unwrapping is straightforward. The unwrapped phase can be evaluated by a simple path-independent integration of the phase differences of adjacent wrapped phases, starting from a reference location and using the assumption that all phase differences are in the interval (-π, π).

In actual interferograms phase unwrapping is more complicated because of phase steps outside the interval (-π, π). Causes for local phase gradients larger than π are:

1. **Phase Noise:** Temporal decorrelation, shadow and low SNR cause phase noise. For repeat-track interferometry temporal decorrelation is often the main cause.
2. **Phase Under-sampling**: The phase is under-sampled when the phase gradient exceeds half a fringe (phase cycle) per sample. In the presence of phase noise the under-sampling already occurs at lower gradients.
3. **Phase Discontinuities:** In interferograms layover and discontinuous surface deformation (e.g. at sliding faults or at glacier rock interfaces) cause discontinuities in the interferometric phase.

2 PHASE UNWRAPPING ALGORITHMS

One group of unwrapping algorithms poses the problem in terms of a solution of a two-dimensional partial differential equation. Among these is the least-squares algorithm [3]. Intrinsically these methods are not developed for discontinuous functions. Consequently large deviations in the unwrapped phase occur in the vicinity of true phase discontinuities relating to physical features such as layover. Another characteristic of these methods is that the output phase can differs by values different than an exact multiple of 2π.

2.1 BRANCH CUT ALGORITHMS

The principle of branch cut algorithms is to restrict the integration through the image to paths with local phase differences in the interval (-π, π). Summing the phase finite differences about short circular paths permits localization of discontinuities in the wrapped phase field. If the sum is non-zero, a so-called “residue” lies in the region. The residue value or “charge” can be positive (+1) or negative (-1) depending on the sign of the sum. Line segments (branches) are drawn between positive and negative residues in a systematic way to function as barriers during the path integration that cannot be crossed. This discharging of residues results in a consistent, path independent solution. The appearance of these cuts and the residues for a typical interferogram are shown in Figure 1. One can observe that most of the residues come in pairs that lie close together. This is due to phase noise that generates a pair of residues. The basic ideas of the *branch-cut* algorithms and a first implementation were presented by Goldstein [4]. Presently used implementations differ in the methodology used to determine the branch cuts.

In our implementation the branches form tree-like networks that have zero net charge. The goal of the algorithm is to connect residues in such a way that minimizes the net length of branches. Construction of a new tree begins by finding an unvisited residue and connecting to its nearest residue neighbor, regardless of sign. A line ("branch cut") is drawn connecting this residue to the neighbor. If the neighbor has not been visited previously, its charge is added to the net sum of charges for the current tree. If the tree is neutral, then the algorithm searches for a unvisited residue and constructs a new tree, otherwise, the regions around all the current tree residues are searched for new tree members. After a residue has been incorporated in a tree it is marked as visited such that its charge is not counted multiple times. The tree-building terminates when all residues in the interferogram have been visited.

Refinements of this algorithm have included modification of the search strategy in areas of low correlation, and the use of distance weighting of the data to optimize the tree structure. In the original implementation, many unnecessary cuts may be drawn that completely wall of areas. Our implementation incorporates existing trees into the current tree directly thereby reducing redundant branches.

In many cases our implementation of the branch-cut algorithm provides a robust and efficient unwrapping solution that works well for images with high correlation. Under certain conditions multi-looking of the interferogram or filtering can facilitate the unwrapping. Especially, when limiting the unwrapping to areas of higher coherence, respectively lower phase noise, the branch-cut solution is rather conservative, with few unwrapping errors.

The spatially incomplete solution is a significant limitation of the branch-cut solution. In the case of longer-time differential interferograms higher coherence is often restricted to relatively small, spatially disconnected urban areas. In such a case the operator supported bridging of the unwrapped phase can be tedious and uncertain. And, from the conceptual point of view, it is a disadvantage that the optimization is done on a local rather than on a global basis.

The demand for global optimization and automated and optimized unwrapping of disconnected areas of high coherence and the available advanced methodology in network flow optimization lead to the development of the minimum cost flow and triangulation network based phase unwrapping solution presented below.

2.2 MINIMUM COST FLOW AND TRIANGULATION

The minimum cost flow problem defines a network consisting of *nodes* and directed *arcs* that connect the nodes. Associated with each of the arcs are a *flow*, a *cost per unit flow*, and an *arc capacity*. Flow moves from *source nodes* to *sink nodes* through the network arcs. Nodes that are neither sources nor sinks are *trans-shipment* nodes. The MCF problem solution gives the optimum flow in each of the arcs that minimize the total cost. The total cost is defined as the sum of costs for all the arc flows. In the MCF paradigm, flows are integer quantities. This problem has been extensively studied and efficient algorithms have been found and presented in the literature (e.g. Ahuja [7]). The application of minimum cost flow (MCF) techniques to phase unwrapping and thereby achieve a global optimization was first presented by Costantini [6].

In our formulation of the phase unwrapping problem in the network flow form, a node is associated with each of the local closed paths used to evaluate the residues. Source nodes are associated with the positive residues and sink nodes with the negative residues. Integer values of arc flow are equivalent to additional multiples of 2π to add to the gradient derived from the wrapped phase. The pixels lie between the network arcs and nodes. When calculating the unwrapped phase by summing along the path, the gradient is adjusted if there is flow in the arc crossing the path of integration.

The cost per unit flow assigned to each arc is a critical parameter in the optimization process. The cost should be low at the place of a real discontinuity. Generally, it should rather be expensive to cause a phase jump in an area of low phase noise, and inexpensive in noisy regions. In our implementation weight factors in the interval (0.0,1.0) can be indicated; for simplicity the costs are calculated in such a way that the coherence or a similar quantity can be used as weight factors.

Another important element for improving the unwrapping has been to generalize the network topology to be a triangulation network as was previously proposed by [8]. In our implementation a Delaunay triangulation is used [9]. Using the triangulation has several distinct advantages. Only those points that have reliable enough phase values are considered in the unwrapping. Areas of too high phase noise are not considered.

As was the intention, the combination of triangulation with the MCF algorithm permits robust phase unwrapping in many cases of isolated areas of high coherence. This is particularly advantageous in the case of long interval differential interferograms. An example of this is shown in Figure 2 where the discontinuous regions were correctly unwrapped without operator interaction. Another advantage of the triangular network is the two times higher density of the network which permits better localization of phase discontinuities leading to more precise unwrapping. The MCF solution is rather memory intensive. A special method was implemented to still permit unwrapping of very large interferograms. Figure 3 shows the use of adaptive thinning the input interferogram to reduce the total number of nodes. Regions of lower correlation and steep slopes have a denser triangle density compared with flat high correlation areas. Patching is another approach to overcome the problem, but the optimization area is reduced to the patch size.

3 CONCLUSIONS

In this paper we have reviewed several practical approaches to phase unwrapping that take into the account the characteristics of SAR interferograms including regions with very high noise and phase discontinuities. The introduction of the MCF and triangulation paradigms significantly improves the robustness and applicability of SAR interferometry to geophysical research. The commercial availability of these algorithms is leading to wide spread application in the user community.

4 REFERENCES

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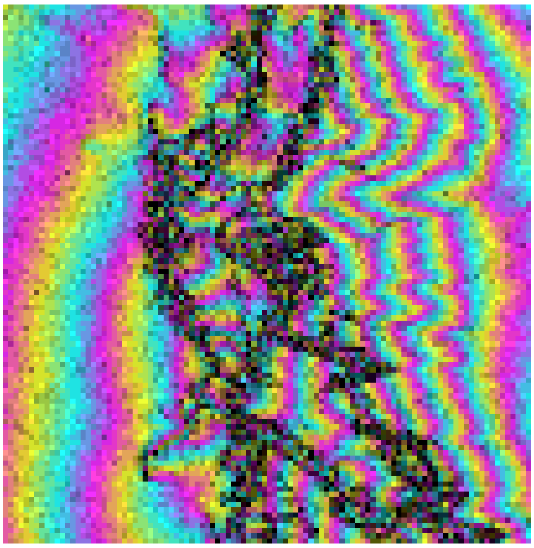


Figure 1a: Interferogram showing interferometric phase intensity weighted by the interferometric correlation. Areas of layover have low correlation and significant phase discontinuities. Phase display is 2π per color cycle.

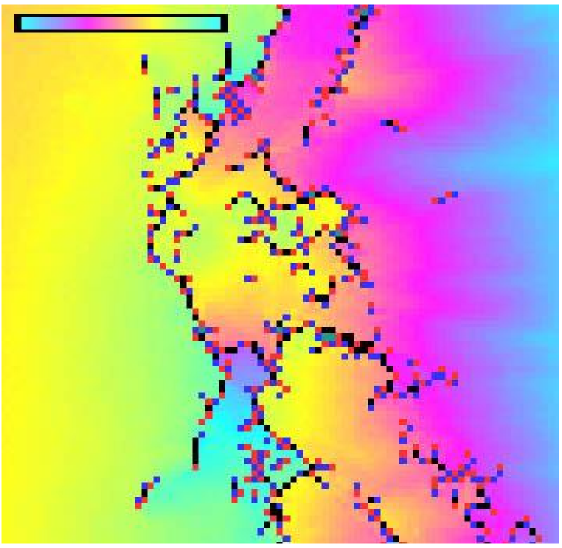


Figure 1b: Unwrapped interferometric phase showing residues (+: red, -: blue) and branch cuts (black). Note that most residues come in +/- pairs. Chains of residues occur along regions of lay-over. Phase display is 10π per color cycle.

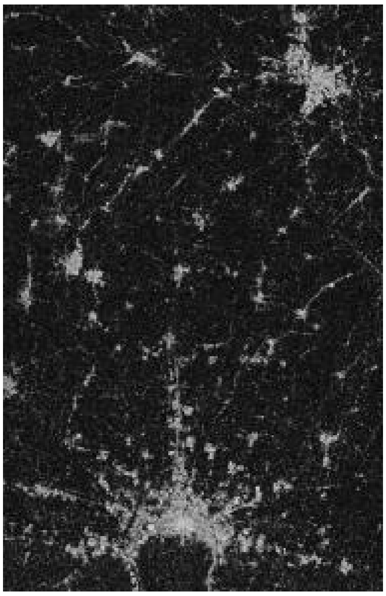


Figure 2a: "Correlation" map for a filtered differential interferogram. A linear scale in the range (0.0, 1.0) is used to display the "correlation" values.

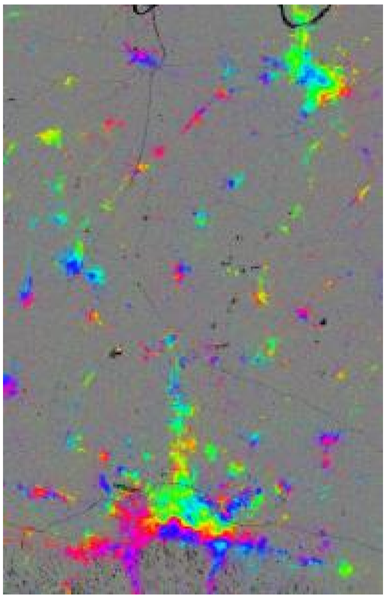


Figure 2b: Interferometric phase after adaptive filtering. Phase displayed as 2π per color cycle.

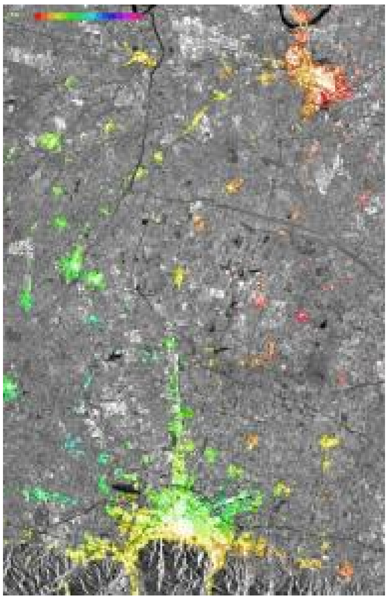


Figure 2c: Unwrapped interferometric phase using triangulation to connect regions. Phase displayed as 10π per color cycle.

Figure 3a: ERS Tandem coherence over Las Vegas. A linear scale in the range (0.0, 1.0) is used to display the "correlation" values.

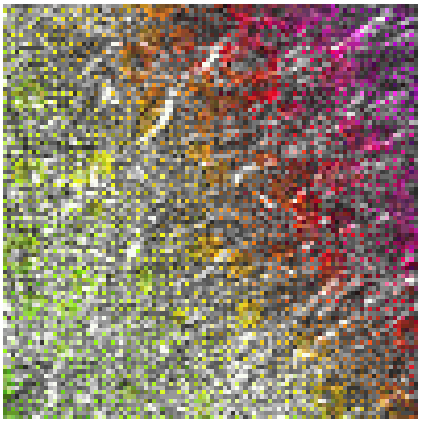


Figure 3b: Sampled unwrapped phase showing adaptively thinned unwrapped phase mesh. Phase displayed as 20π per color cycle.

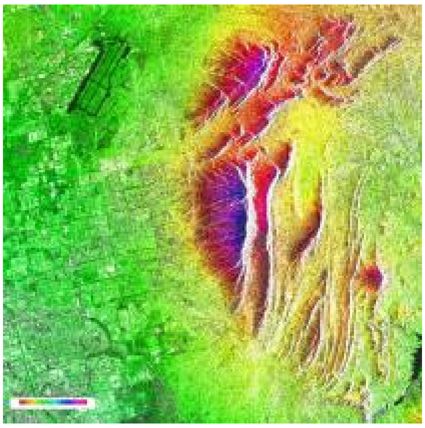


Figure 3c: Full unwrapped phase using sampled unwrapped data as model for the full interferogram. Phase displayed as 20π per color cycle.

Appendix 1: Pre-Processing

INTRODUCTION

Pre-processing as we use it in this context is understood as steps in preparation of phase unwrapping. The pre-processing results in the input data sets for the phase unwrapping. Strictly speaking the phase unwrapping sequence generates an unwrapped phase image for the complex valued data set which contains the wrapped phases. Re-wrapping of the unwrapped phase should result again in the wrapped phase of the input data set. Consequently, steps which modify the phase by values different from a multiple of 2π are not considered as phase unwrapping steps but as pre- or post-processing steps. A typical pre-processing step is filtering. The distinction between steps which are part of the pre-processing and unwrapping steps is not very sharp, though, because of the high flexibility and modularity of the approach. Multi-looking, for example, can also be considered as a part of the phase unwrapping if the unwrapped image in the multi-look geometry is used as phase model to unwrap the full resolution data.

The purpose of the pre-processing steps is to facilitate the phase unwrapping. This can be done by reducing the phase noise (multi-looking, filtering), by reducing the size of the data set (multi-looking, masking), and by reducing the complexity of the phase function through subtraction of a phase model (DINSAR). Furthermore, the estimation of the coherence or a similar measure for the phase noise is included in the pre-processing steps, as this measure may be used as a weighting function in the phase unwrapping

MULTI-LOOKING

Multi-looking of the complex valued data set which contains the wrapped phases is a very important step to facilitate phase unwrapping. The main advantages of multi-looking are highly effective reduction of the phase noise, increased efficiency, and ability to unwrap very large data sets, with the last two resulting from the reduction of the data set sizes. Multi-looking is particularly adequate when unwrapping data sets which are at high spatial resolution such as 5-look ERS interferograms and 3-look JERS interferograms.

The main limitation of multi-looking is that the reduction in the sampling may cause under-sampling in the case of higher phase gradients. Consequently, multi-looking is generally more appropriate for relatively smooth phase surfaces.

Multi-looking of complex and real valued data sets is supported by the ISP programs *multi_cpx* and *multi_real*. The same programs also support the inverse operation, i.e. the expansion of a data in the multi-looked geometry into the original geometry. This permits, for example, to apply a 2x multi-looking in range and in azimuth direction, to unwrap the multi-looked data set, and then to expand the unwrapped phases again to the original geometry. This expanded unwrapped phase can then be used as phase model to unwrap the data set at full resolution, which is supported by the ISP program *unw_model*.

FILTERING

The objective of filtering is to reduce phase noise but to maintain the spatial sampling. In the design of the filter function it should be considered that the interferometric phase cannot be assumed to be constant across the filter window.

Successful filtering results in a strong reduction of the number of residues which can reduce the complexity of the phase unwrapping problem and which can increase the phase unwrapping efficiency. Apart from the effect on the phase unwrapping another important reason for filtering is the actual reduction of the noise of the interferometric phase. Filtering of the complex valued data set is an attractive alternative to filtering of the real valued unwrapped phase, respectively of derived parameters as the terrain height or surface deformation.

Limitations of filtering include unwanted effects in the case of phase discontinuities (e.g. layover) and high phase gradients (loss of fringes) which may lead to phase unwrapping errors, and the increasing spatial dimension of the granular phase variation in the case of dominant noise.

Low-pass filtering is adequate for cases where the phase gradients are overall small. In the GAMMA ISP low-pass filtering is supported by the band-pass filtering program *bpf*. Alternatives, supported in the GAMMA ISP are a filter which is adaptive to the local phase slope (*adapt_filt*) and a filter which applies a non-linear spectral filter with a filter function which is a function of the power spectrum (*adf*).

DINSAR

The phase unwrapping problem in SAR interferometry is closely related to the co-existence of complex phase functions and phase noise. As complex phase functions we understand phase functions with high phase gradients and even discontinuities. For the wrapped phase image this results in under-sampling for parts of the image. Multi-looking and filtering address the reduction of the phase noise, which can be done quite successfully in the case of relatively simple phase functions. The objective of using differential interferometric techniques as a pre-processing step to phase unwrapping is to reduce the complexity of the phase image. Subtracting a simulated topographic phase component from an interferogram of a hilly terrain typically permits to reduce the residual phase values to a small number of phase cycles. One clear advantage is, that stronger multi-looking and filtering may be applied to a differential interferogram with a relatively smooth phase surface. The resulting differential interferogram can be expected to be simpler to unwrap, which is in practice often the case, but not always.

A complete set of tools for differential interferometric processing is part of the GAMMA DIFF&GEO package.

COHERENCE AND PHASE NOISE

The degree of coherence is defined as the magnitude of the normalized interferogram **γ**, defined as the normalized complex correlation coefficient of the complex backscatter intensities s_1 and s_2

$$\gamma = \left| \frac{\langle s_2 s_1 {}^* \rangle}{\sqrt{\langle s_1 s_1 {}^* \rangle \langle s_2 s_2 {}^* \rangle}} \right|,$$

with the brackets $\langle x \rangle$ standing for the ensemble average of x. The variance of the estimate of the interferometric phase $\hat{\phi}$ is reduced by coherent averaging over a set of looks, which are statistically independent samples of the resolution element. Under the assumption that the N looks have the same statistics the maximum-likelihood estimator of the phase becomes:

$$\hat{\phi} = a\tan\left(\frac{\Im(\sum_{k=1}^{N} s_2(k) s_1{}^*(k))}{\Re(\sum_{k=1}^{N} s_2(k) s_1{}^*(k))} \right).$$

The Cramer-Rao bound on the standard deviation of the interferometric phase σ_ϕ (reached asymptotically for large number of looks N) is a function of the degree of coherence,

$$\sigma_\phi = \frac{1}{2N} \frac{\sqrt{1-\gamma^2}}{\gamma}.$$

The coherence depends on radar system and data processing parameters, geometric parameters, and parameters related to the target. Random dislocation of the individual scatterers between the two acquisitions of an interferometric image pair reduces the degree of coherence.

The basic philosophy of the GAMMA software concerning coherence estimation is that the estimate should be as much as possible independent of processing parameters. The same coherence shall be estimated independent of the multi-looking used for the interferogram calculation. Consequently, coherence should never be estimated from a filtered interferogram.

In the context of phase unwrapping we are not primarily interested in the above defined coherence but in a measure for the phase noise of the interferogram we want to unwrap. Filtering has an effect on the phase noise. A parameter to characterize the phase noise of a filtered interferogram is obtained by applying the coherence estimation program to the filtered interferogram. This measure is adequate for further use in the phase unwrapping; sometimes we call it “effective coherence” or “coherence” but it should not be called coherence. The advantage of this measure is the more direct characterization of the phase noise of the actual interferogram to be unwrapped. The disadvantage is that such values depend on the processing which makes it necessary to more carefully verify thresholds etc. used in the unwrapping.

Appendix 2: Phase Unwrapping using Branch-Cut Algorithm

INTRODUCTION

The principle of branch cut algorithms is to restrict the integration through the image to paths with local phase differences in the interval (-π, π). Summing the phase finite differences about short circular paths permits localization of discontinuities in the wrapped phase field. If the sum is non-zero, a so-called "residue" lies in the region. The residue value or "charge" can be positive (+1) or negative (-1) depending on the sign of the sum. Line segments (branches) are drawn between positive and negative residues in a systematic way to function as barriers during the path integration that cannot be crossed. This discharging of residues results in a consistent, path independent solution.

PROCESSING STEPS

Pre-processing

Pre-processing may include steps to reduce the complexity of the problem and phase filtering. For more discussion of the different possibilities it is referred to Appendix 1.

Masking low correlation areas

Areas of low "coherence" (i.e. below a user-specified threshold) are not included in the phase unwrapping due to too high phase noise. Such areas of low correlation are masked using the program *corr_flag*. A phase unwrapping flag file is generated containing the low correlation mask as well as further information which will be determined later on.

Neutrons

Neutrons may be used to guide branch cuts through regions of layover and to exclude layover from unwrapping (*neutron*). The use of neutrons is optional. Neutrons are treated by the unwrapping code as residues with no charge. Therefore, these points are included in branch cuts but do not contribute to the net charge of a tree. Adding neutrons based on image intensity will cause lay-over to be excluded from unwrapping because a dense network of branch cuts will occur if any charged residues are present. The layover area is detected simply by backscattering which is more than a user specified factor higher than the average backscattering. The neutrons are written to the phase unwrapping flag file.

Determination of residues

Residues are locations around which a closed integral of the phase differences (assuming phase differences are in the interval [-π,π]) leads to a non-zero result. Such location lead to inconsistencies in the phase unwrapping process. Residue locations are determined using one of the programs *residue_cc* and *residue* and are written to the phase unwrapping flag file. *residue_cc* differs from *residue* in that the phases of pixels in the masked (low coherence) areas are assumed to be zero which significantly reduces the total number of residues while maintaining the total charge of the area.

Connection of residues through neutral trees

In a next step positive and negative residues are "discharged" through "neutral trees". During the unwrapping of the interferometric phase no neutral trees will be crossed. Therefore, the result will be independent of the path chosen for the unwrapping.

In our two implementations (*tree_gzw*, *tree_cc*) the branches form tree-like networks that have zero net charge. The goal of the algorithms is to connect residues in such a way that minimizes the net length of branches. Construction of a new tree begins by finding an unvisited residue and connecting to its nearest residue neighbor, regardless of sign. A line ("branch cut") is drawn connecting this residue to the neighbor. If the neighbor has not been visited previously, its charge is added to the net sum of charges for the current tree. If the tree is neutral, then the algorithm searches for a unvisited residue and constructs a new tree, otherwise, the regions around all the current tree residues are search for new tree members. The tree-building terminates when all residues in the interferogram have been visited.

Refinements of the algorithm have include modification of the search strategy in areas of low correlation, and the use of distance weighting of the data to optimize the tree structure. In the original implementation, many unnecessary cuts may be drawn that completely wall of areas. *tree_cc* incorporates exiting trees into the current tree directly thereby reducing redundant branches.

Unwrapping of interferometric phase

The phase unwrapping is started at a user-defined location (default location is the image center) and continued by region growing for the entire area connected to the starting location avoiding low correlation areas and without crossing neutral trees (*grasses*). The phase unwrapping flag file contains the information which area was unwrapped. The unwrapped interferometric phase is written to a real valued data file.

Construction of bridges between disconnected areas

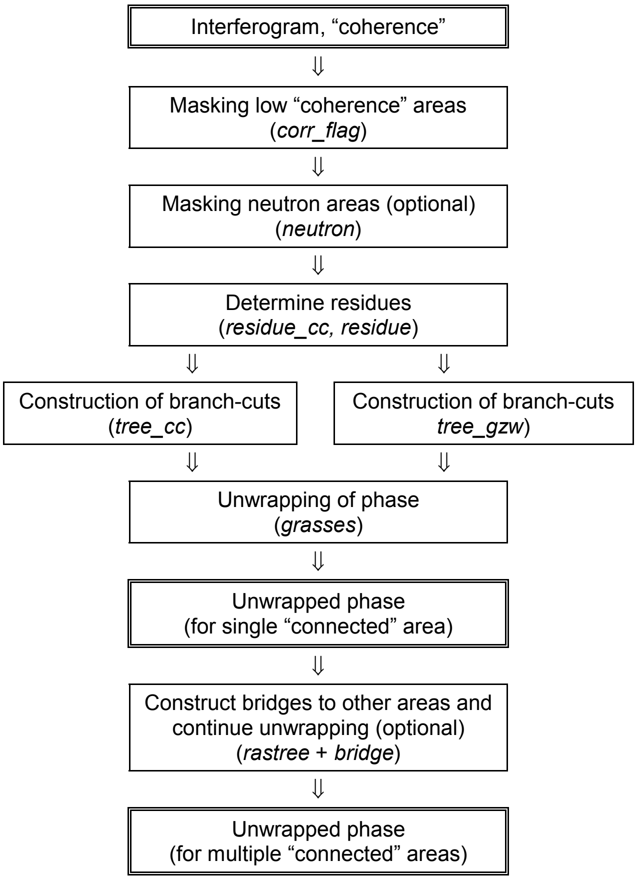
The unwrapping may be continued in areas disconnected from the already unwrapped area by constructing bridges between unwrapped and not unwrapped locations. Very often the two locations connected by the bridge are assumed to be on the same ambiguity. If that is not the case the desired multiple of 2π difference can be indicated. To construct the bridges a SUN rasterfile is generated (*rastree*) to display the wrapped phases (darker colors) together with the already unwrapped phases (brighter colors) which allows to edit the "bridges file", an ASCII file with the content as shown below. The bridges are then used to unwrap the phase in the disconnected areas. The same procedure can be repeated with new bridges to unwrap further disconnected areas.

Unwrapped location		Wrapped location		Offset
column	row	column	row	in 2π
458	913	430	923	0
612	405	655	398	1

Example of "brides file" with two bridges.

Post-processing

Post-processing may include operator correction of the flag file to avoid phase unwrapping errors which occurred, filtering of the unwrapped phase, and interpolation of the unwrapped phase to fill in gaps. For more discussion of the different possibilities it is referred to Appendix 4.



Flow chart for Phase Unwrapping using Branch-Cut Algorithm. *Italic font* refers to GAMMA ISP program names

EXAMPLE FOR PHASE UNWRAPPING USING BRANCH-CUT ALGORITHM

The task is to unwrap the flattened interferogram a_b.flt.

As pre-processing steps the interferogram is adaptively filtered and an "effective correlation", a_b.flt.sm.cc, is estimated :

adf a_b.flt a_b.flt.sm a_b.flt.sm.cc 750 .3

The filtered interferogram has the same format as the unfiltered interferogram. Typical exponent parameters for *adf* lie in the range of .2 to .5. For the phase unwrapping it is necessary to have an estimate of the local fringe quality after filtering. Because of the degree of filtering varies over the image, *adf* estimates the local phase standard deviation after filtering and converts this to an "effective correlation" which will then be used in the unwrapping.

The next steps are to generate the unwrapping flag file and mask areas of low "effective correlation", to add neutrons, and to determine and add the residues. Then the branch-cuts are determined (using *tree_cc* or *tree_gzw*) and the phase is unwrapped without crossing any branch-cuts starting at a user-defined location (default location is the image center) and continued by region growing for the entire area connected:

corr_flag a_b.cc a_b.flt.sm.flag 750 0.25

neutron a_b.pwr1 a_b.flt.sm.flag 750 6.0

residue a_b.flt.sm a_b.flt.sm.flag 750

tree_cc a_b.flt.sm.flag 750

grasses a_b.flt.sm a_b.flt.sm.flag a_b.flt.sm.unw 750

The real valued data file a_b.flt.sm.unw contains the unwrapped interferometric phase which can be displayed, for example, using disrmg.

To unwrap the unfiltered interferogram a_b.flt.sm.unw can be used as a model:

unw_model a_b.flt a_b.flt.sm.unw a_b.flt.unw 750

In the same step a phase offset can be introduced to set the phase at a reference location to an indicated reference value.

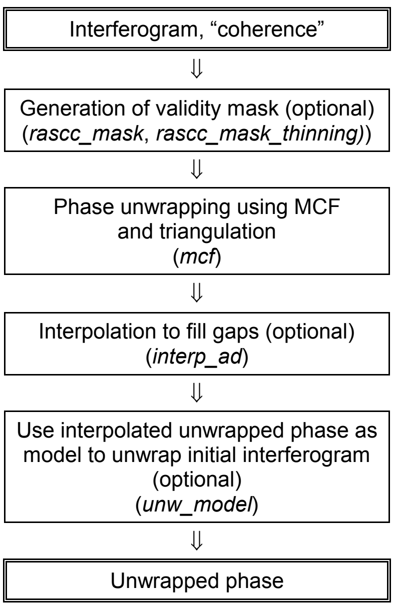
Appendix 3: Phase Unwrapping using MCF Algorithm

INTRODUCTION

The demand for global optimization and automated and optimized unwrapping of disconnected areas of high coherence and the available advanced methodology in network flow optimization lead to the development of the minimum cost flow (MCF) and triangular network based phase unwrapping solution.

In our formulation of the phase unwrapping problem in the network flow form, a node is associated with each of the local closed paths used to evaluate the residues. Source nodes are associated with the positive residues and sink nodes with the negative residues. The network is not a rectangular network but an triangular irregular network (TIN) which is more flexible, for example in the case of disconnected areas of high coherence. The pixels lie between the network arcs and nodes. When calculating the unwrapped phase by summing along the path, the gradient is adjusted if there is flow in the arc crossing the path of integration.

The cost per unit flow assigned to each arc is a critical parameter in the optimization process. The cost should be low at the place of a real discontinuity. Generally, it should rather be expensive to cause a phase jump in an area of low phase noise, and inexpensive in noisy regions. In our implementation weight factors in the interval (0.0,1.0) can be indicated; for simplicity the costs are calculated in such a way that the coherence or a similar quantity can be used as weight factors.



Flow chart for Phase Unwrapping using MCF Algorithm. *Italic font* refers to GAMMA ISP program names. If no validity mask is used the phase unwrapping is straight forward using the single ISP program *mcf*.

PROCESSING STEPS

Pre-processing

Pre-processing may include steps to reduce the complexity of the problem, for phase filtering, and to reduce the data volume to a size which permits global optimization. For more discussion of the different possibilities it is referred to Appendix 1.

Generation of validity mask

The phase unwrapping validity mask is used as a tool to mask areas where the phase values shall not be considered in the MCF optimization. The validity mask is a SUN raster or BMP file with NULL (0,0,0) values for pixels which shall not be considered in the MCF phase unwrapping. It is generated using the program *rascc_mask*. A file of type float, for example a coherence image, is used and values below an indicated threshold are set to NULL (0,0,0) in the output rasterfile. The use of a validity mask is optional. To use a validity mask is recommended for data with predominantly high phase noise, as in the case of long-term differential interferograms.

One advantage of using a validity mask is that the number of values considered in the unwrapping is reduced. This reduction can be further increased by reducing the sampling (*rascc_mask_thinning*). This "thinning" of the points considered in the phase unwrapping is done adaptively to the phase variation (noise, gradient). In areas of low phase variation sparser sampling is possible, which allows to increase the efficiency of the MCF phase unwrapping.

The masking and adaptive sampling reduction can result in a significant reduction of the number of samples which have to be considered in the MCF optimization. Apart from the gain in efficiency this also allows to apply real global optimization (i.e. without patch processing) for relatively large interferograms.

Phase unwrapping using MCF and triangulation

Phase unwrapping using a triangular network and MCF optimization is done using the program *mcf*. A validity mask and a weighting function can be provided.

In the case of complete data sets without gaps the "filled triangulation mesh", in other cases the "Delaunay triangulation" should be selected for the definition of the triangular network.

Processing in a single patch is preferred for the global optimization done in this case. If the indicated memory requirement is larger than the available memory of the computer it is recommended to reduce the size of the patches, otherwise the optimization may be very time consuming (if swap space is used). In this case the processing and optimization is done in multiple patches.

Interpolation to fill gaps

The gaps in the unwrapped phase image can be filled by interpolation. An interpolator with an adaptive window size is used (*interp_ad*), i.e. for the filling of small gaps a small window is used, for the filling of larger gaps a larger window. This allows to avoid unwanted filtering as would occur with a constant, and therefore relatively large, window size.

Use interpolated unwrapped phase as model to unwrap initial interferogram

The unwrapped phases are recalculated from the complex valued interferogram (*unw_model*) assuming that the phase values in the model correspond to the correct unwrapped phase within the interval ($-\pi,\pi$). The resulting unwrapped phase meets the condition that re-wrapping of the unwrapped phase results in exactly the phase of the complex interferogram, except for a constant offset which can be defined through the phase indicated for the reference location.

Post-processing

Post-processing may include filtering of the unwrapped phase, and interpolation of the unwrapped phase to fill in gaps. For more discussion of the different possibilities it is referred to Appendix 4.

EXAMPLES FOR PHASE UNWRAPPING USING MCF ALGORITHM

Interferogram with generally high coherence

A possible processing sequence for the unwrapping of an interferogram with generally high coherence such as used in the generation of an interferometric DEM from ERS Tandem data, consists of a 2x2 multi-looking of the 5-look complex valued interferogram, the coherence estimation in multi-looked interferogram, the MCF phase unwrapping step using a "filled triangulation mesh", multiple patches and the coherence as weights, and the expansion of the unwrapped phase image to the original geometry:

```
multi_cpx a_b.flt a_b.off a_b.flt2 a_b.off2 2 2
cc_wave a_b.flt2 - - a_b.cc2 2456 5 5
mcf a_b.flt2 a_b.cc2 - a_b.unw2 2456 0 0 0 - - 4 4 256
multi_real a_b.unw2 a_b.off2 a_b.unw a_b.off -2 -2
```

In the case of unwrapping errors filtering of the multi-looked interferogram and using a validity mask are the first attempts to resolve the problem.

Long-term differential interferogram with isolated islands of high coherence

A possible processing sequence for the unwrapping of a long-term differential interferogram with only relatively small isolated islands of high coherence, consists of a adaptive filtering and estimation of an "effective correlation", generation of a validity mask, and the MCF phase unwrapping step using a "Delaunay triangulation", a single patch and the "effective coherence" as weights:

```
adf a_b.flt a_b.flt.sm a_b.flt.sm.cc 2100 .3 64 7 2
rascc_mask a_b.flt.sm.cc a_b.pwr 2100 1 1 0 1 1 0.25 0.1 0.9 1. .35 1 a_b.mask.ras
mcf a_b.flt.sm a_b.flt.sm.cc a_b.mask.ras a_b.flt.sm.unw 2100 1
```

In the case of unwrapping errors modifications to the filtering (for example multiple application of adf) and to the masking are the first attempts to resolve the problem. In the case of insufficient memory adaptive thinning may be used to maintain global optimization.

Appendix 4: Post-Processing

INTRODUCTION

Post-processing as we use it in this context is understood as steps applied after the primary phase unwrapping sequences described in Appendices 2 and 3. The main purposes of post-processing steps are to modify/enhance results of unwrapping sequence and to undo effects of pre-processing transformations.

UNWRAPPING ERROR CORRECTION

The program *tree_edit* is a tool to interactively modify phase unwrapping flag files as used in the branch-cut approach (see Appendix 2). Obvious phase unwrapping errors, i.e. phase jumps of approximately 2π in areas where no phase discontinuity is expected, can be avoided by modification of the cuts. Such operator interaction should be limited to a few corrections.

MULTI-LOOKING

Multi-looking done in the pre-processing to facilitate the phase unwrapping (smaller data volumes, less phase noise, higher efficiency) can be compensated for by expanding the data set again to the initial geometry. The same programs *multi-cpx* and *multi_real* are used, but with negative multi-looking factors (-2 to undo a multi-looking with a factor 2).

FILTERING, INTERPOLATION

The real valued unwrapped phase image can be low-pass filtered using the band-pass filter *bpf*. Low-pass filtering is applied to reduce phase noise.

Gaps in the unwrapped phase image can be filled by interpolation. An interpolator with an adaptive window size is used (*interp_ad*), i.e. for the filling of small gaps a small window is used, for the filling of larger gaps a larger window. This allows to avoid unwanted filtering as would occur with a constant, and therefore relatively large, window size.

UNWRAPPING USING A PHASE MODEL

Phase unwrapping based on a phase model assuming that the phase values in the model correspond to the correct unwrapped phase within the interval $(-\pi,\pi)$ is a (post-)processing tool with a relatively wide application range. An unwrapped filtered or multi-looked interferogram can be used as a model to unwrap the original (unfiltered, not multi-looked) interferogram. Interpolated unwrapped phase values can be used as model to get the “real” unwrapped values.

The resulting unwrapped phase meets the condition that re-wrapping of the unwrapped phase results in exactly the phase of the complex interferogram, except for a constant offset which can be defined through the phase indicated for the reference location.