Characterization of Differential Interferometry Approaches

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Abstract

Different approaches of 2-, 3-, and 4-pass differential interferometry are presented and its potential and limitations are discussed. The focus is on the methodology and not on a specific application example.

1. Introduction

In recent years space-borne repeat-pass differential SAR interferometry has demonstrated a good potential for displacement mapping with sub-cm resolution. Applications exist in the mapping of seismic and volcanic surface displacement as well as in land subsidence and glacier motion.

The interferometric phase is sensitive to both surface topography and coherent displacement along the look vector occurring between the acquisition of the interferometric image pair. Inhomogeneous propagation delay ("atmospheric disturbance") and phase noise are the main error sources. The basic idea of differential interferometric processing is to separate the topography and displacement related phase terms. Subtraction of the topography related phase leads to a displacement map. In the so-called 2-pass differential interferometry approach the topographic phase component is calculated from a conventional Digital Elevation Model (DEM). In the 3-pass and 4-pass approaches the topographic phase is estimated from an independent interferometric pair without differential phase component. In practice, the selection of one of these approaches for the differential interferometric processing depends on the data availability and the presence of phase unwrapping problems which may arise for rugged terrain.

In the case of stationary motion the displacement term may be subtracted to derive the surface topography. A typical application of this technique is the mapping of the surface topography of glaciers.

2. Interferometry

Prior to the discussion of differential interferometry approaches a few facts on SAR interferometry are reviewed.

The unwrapped phase ϕ_{unw} of an interferogram can be expressed as a sum of a topography related term ϕ_{topo}, a displacement term ϕ_{disp}, a path delay term ϕ_{path}, and a phase noise (or decorrelation) term ϕ_{noise}:

$$\phi_{unw} = \phi_{topo} + \phi_{disp} + \phi_{path} + \phi_{noise} \qquad (1)$$

The baseline geometry and ϕ_{topo} allow to calculated the exact look angle and together with the orbit information the 3-dimensional position of the scatter elements (and thereby the surface topography).

The displacement term, ϕ_{disp}, is related to the *coherent* displacement of the scattering centers along the radar look vector, r_{disp}:

$$\phi_{disp} = 2kr_{disp} \qquad (2)$$

where k is the wavenumber. Here *coherent* means that the same displacement is observed of adjacent scatter elements.

Changes in the effective path length between the SAR and the surface elements as a result of changing permittivity of the atmosphere, caused by changes in the atmospheric conditions (mainly water vapor), lead to non-zero ϕ_{path}.

Finally, random (or incoherent) displacement of the scattering centers as well as noise introduced by SAR signal noise is the source of ϕ_{noise}. The standard deviation of the phase noise σ_ϕ (reached asymptotically for large number of looks N) is a function of the degree of coherence, γ [1],

$$\sigma_\phi = \frac{1}{2N}\frac{\sqrt{1-\gamma^2}}{\gamma}. \qquad (3)$$

Multi-looking and filtering allow to reduce phase noise. The main problem of high phase noise is not so much the statistical error introduced in the estimation of ϕ_{topo} and ϕ_{disp} but the problems it causes with the unwrapping of the *wrapped* interferometric phase. Ideally, the phase noise and the phase difference between adjacent pixels are both much smaller than π. In reality this is often not the case, especially for areas with a low degree of coherence combined with rugged topography, as present in the case of forested slopes.

Assuming that there is no surface displacement, i.e. $\phi_{disp} = 0$, allows to relate ϕ_{unw} to surface topography, with ϕ_{noise} introducing a statistical error and ϕ_{path} introducing a non-statistical error. In a similar way assuming that $\phi_{topo} = 0$ allows to interpret ϕ_{unw} as ϕ_{disp} which can be related to coherent surface displacement along the look vector, again with ϕ_{noise} introducing a statistical error and ϕ_{path} introducing a non-statistical error. It is important to keep in mind that the topography related phase term gets small not only for negligible surface topography but also for very small $B_\perp$ due to its indirect proportionality with the baseline component perpendicular to the look vector $B_\perp$.

The main objective of differential interferometry is the isolation of the surface topography and the surface displacement contributions to the unwrapped interferometric phase, including all the more general cases with $\phi_{disp} \neq 0$ and $\phi_{topo} \neq 0$.

3. Wrapping operator

In the selection of the most appropriate differential interferometry approach for a specific case phase unwrapping plays an important role. For this reason the *wrapping operator* ***W*** is discussed here. In a complex interferogram the interferometric phase is only known modulo 2π, i.e. the values are all "projected" into the base interval, which we select to be [-π,π]. To avoid confusion we use the wrapping operator ***W*** to indicate wrapped phases, i.e. $W[\phi]$ is the wrapped phase of the "unwrapped" phase ϕ. As a support for the calculation with wrapped phase terms we write up some rules of the algebra of the wrapping operator. For the addition the following identity is valid:

$$W[\phi_1 + \phi_2] = W\left[W[\phi_1] + W[\phi_2]\right] \qquad (4)$$

As a consequence addition or subtraction of phase images can be done before or after phase unwrapping, or in other words unwrapping the sum of wrapped phase components is equal to the sum of the unwrapped phases. The wrapping operator has also to be applied to the sum as the sum may exceed 2π.

For the multiplication with a real valued factor *k* we find:

$$W[k\phi] = W\left[kW[\phi]\right] \quad k \in I \text{ (Integer)} \qquad (5)$$
$$W[k\phi] \neq W\left[kW[\phi]\right] \quad k \notin I$$

For interferometry this means that unwrapping of a scaled wrapped phase is identical to the scaled unwrapped phase only for integer scaling factors. As a consequence the scaling of the topographic phase to account for differences in the perpendicular baseline component can only be done for the unwrapped phase image. Nevertheless, scaling of a wrapped phase image with an integer scale factor can be used as an approximation if phase unwrapping could not be performed.

4. Differential Interferometry

4.1. Displacement mapping

The objective in displacement mapping is to isolate the displacement phase term.

4.1.1. Negligible topographic phase

For the case of a very small topographic phase component the interferometric phase can directly be interpreted as displacement phase. This simpler case is found a) for almost flat surfaces and b) for interferograms with very small perpendicular baseline components.

The relation between a change in the topographic height σ_h and the corresponding changes in the interferometric phase σ_ϕ is given by,

$$\sigma_h = \frac{\lambda r_1 \sin\theta}{4\pi B_\perp}\sigma_\phi. \qquad (6)$$

For the ERS-1 and ERS-2 SAR sensors, with a wavelength is 5.66cm, a nominal incidence angle of 23 degrees, and a nominal slant range of 853 km Equation (6) reduces to

$$\sigma_h \approx 1500\frac{\sigma_\phi}{B_\perp[m]}, \qquad (7)$$

allowing to estimate the effect of the topography.

4.1.2. General case

For the case of $\phi_{topo} \neq 0$ it is necessary to estimate and subtract ϕ_{topo}. The estimation, $\phi_{topo,est}$, differs from ϕ_{topo} by the estimation error $\phi_{topo,error}$:

$$\phi_{topo,est} = \phi_{topo} + \phi_{topo,error} \quad (8)$$

An estimate $\phi_{disp,est}$ of the displacement phase term, ϕ_{disp} is then found by subtraction of $\phi_{topo,est}$ from ϕ_{unw}

$$\phi_{disp,est} = \phi_{unw} - \phi_{topo,est} \quad (9)$$

So far we assumed that all the phase terms are available in its unwrapped form. It may be that only wrapped interferometric phase $W[\phi_{unw}]$ is known. The topographic phase term may be estimated either based on a digital elevation model (DEM) (Fig. 1, methods 1,2) or an independent interferogram without displacement (Fig. 1, methods 3-5). The derivation based on a DEM allows to directly estimate the unwrapped topographic phase term $\phi_{topo,est}$. The estimation from an independent interferogram starts from its wrapped interferometric phase. Here we can further distinguish between two cases based on the criteria if we succeed in unwrapping this wrapped phase. For the estimation of the topographic phase term of the reference interferogram 1, $\phi_{1,topo,est}$, the topographic phase term of the interferogram 2, $\phi_{2,topo}$, needs to be scaled by the ratio between the perpendicular baseline components

$$\phi_{1,topo,est} = \phi_{offset} + \frac{B_{1\perp}}{B_{2\perp}} \phi_{2,topo} \quad (10)$$

In general the ratio $B_{1\perp} / B_{2\perp}$ is not an integer and therefore the precise scaling cannot be done without phase unwrapping. In cases where neither a DEM is available nor phase unwrapping of the topographic reference interferogram was successful the scaling of the wrapped phase images with integer factors may provide the best result (Fig. 1, method 5). For $B_{1\perp} = 100m$ and $B_{2\perp} = 183m$, for example, the wrapped differential interferogram calculated as

$$W[\phi_{diff}] = W\left[2 \cdot W[\phi_1] - W[\phi_2]\right] \quad (11)$$

contains twice the displacement phase term but just a very small topographic phase term corresponding to a baseline of -17*m*. It has to be kept in mind though, that the scaling will also scale the phase noise.

4.2. Topographic mapping

4.2.1. Negligible displacement phase

In many cases surface displacement can be neglected, i.e. it can be assumed that $\phi_{disp} = 0$. The interferometric phase can directly be used as the estimate of the topographic phase term; no differential interferometric processing is required.

4.2.2. General case

In the general case there are non-negligible topographic and differential phase terms. The displacement term does not depend on the baseline. Under the assumption of uniform surface motion, differential interferometry allows to eliminate the displacement phase term and isolate the topography phase term . The phase difference image corresponds to the topographic phase term of an interferogram with a baseline corresponding to the difference between the two baseline vectors. The calculation of the phase difference does not require scaling with non-integer factors and can therefore also be done with wrapped phases.

In the case of glacier motion the assumption of uniform surface motion allowed to estimate the surface topography which is of interest for the study of the ice mass balance [2].

4. Baseline refinement

The topographic phase component is closely related to the interferometric baseline, in particular its component perpendicular to the look vector. As a consequence the exact knowledge of the baselines is very important for the scaling of the topographic phase. Usually, the accuracy of the state vectors does not allow to achieve the required accuracy in the baseline estimation.

One of the strategies we follow is to refine the baseline estimates of the individual interferograms (entitled baseline refinement (a) in Figure 1). This is done based on the interferometric data under the assumption that the topographic phase (including the flat earth phase trend) dominates the signal for the selected area. Either selected control points with known surface heights (ideally in stable areas of the image without differential effects), or the assumption that the imaged area is flat over a large scale are used.

While, phase unwrapping is required for the first method this is not necessary for the second method.

In our second strategy we do not try to optimize the individual baseline estimates but the phase scaling function is directly optimized with a low order fit between the two phase images (entitled *baseline refinement (b)* in Figure 1). This method provides good results as long as the topographic phase (including the phase ramp for the flat Earth) dominates the signal, which is the case as long as the baselines are extremely small.

5. Summary

Differential interferometric approaches were presented and its applicability, advantages and limitations discussed. The selection of an optimal approach for a specific case depends on the INSAR configuration for the available data (baseline, degree of coherence), the availability of a DEM, and the site topography (flat/low/rugged). Examples of processed data will be shown during the presentation.

6. References

[1] Rodriguez E. and J. M. Martin, Theory and design of interferometric synthetic-aperture radars, IEE Proceedings F, vol. 139, no. 2, pp. 147--159, April, 1992.

[2] Joughin I. R., D. Winebrenner, M. Fahnestock, R. Kwok, and W. Krabill, Measurement of ice-sheet topography using satellite radar interferometry, J. Glaciology, vol. 42, no. 140, pp. 10 -22, 1996.

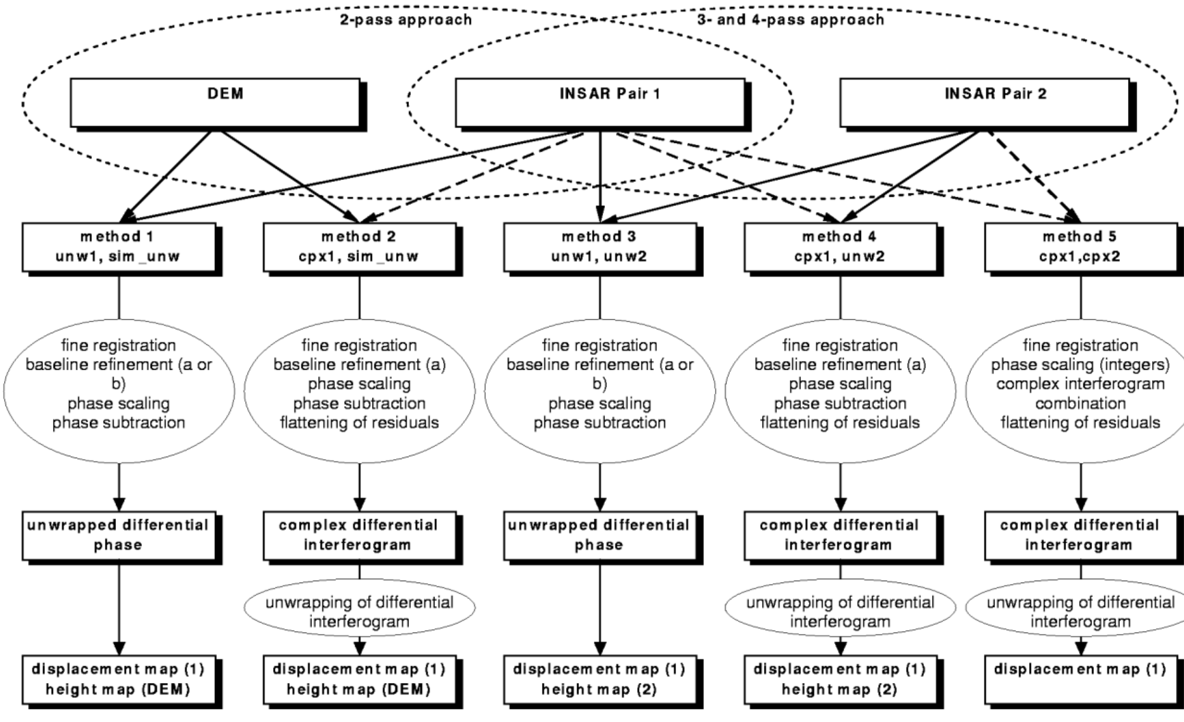

Figure 1 Flow chart for differential interferometric processing methods. The selection of the appropriate method depends on the availability of a digital elevation model (DEM) and on the capability to unwrap the interferometric phase (cpx stands for the complex interferograms, unw for the unwrapped phase image, sim_unw for the simulated unwrapped topographic phase calculated from the DEM). The processing chains for 3- and 4-pass differential interferometry are identical except that no additional registration step is required in the 3-pass approach if both interferometric pairs use the same scene as reference geometry.